Enhancing Performance on Mathematics Word Problems for Students With and Without Mathematics Learning Disabilities

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What's This Session About?

- Math Problem Solving

- RTI

- Instructional Alignment

Math Problem Solving (MPS): The Challenge

- We view MPS as:
  - A form of transfer
  - Requiring students to apply problem-solution rules they have learned to novel problems.

- MPS can be difficult to promote, even among older, more competent learners.

Hell's library

By providing explicit instruction at 2nd and 3rd grades,

we hope to decrease long-term difficulty.

Schema Theory

- Provides our conceptual framework.

- Students use schemas to group problems that require similar solution methods.

- With broader schemas, students more readily recognize that novel problems belong to a problem type for which they know a solution method.

- Schema recognition reduces cognitive load, facilitating problem solution.

Schema Theory: Transfer Features

- Novel problems differ from taught problems in terms of *transfer features*.

- Transfer features alter problems without changing the methods for problem solution.

- The most basic transfer feature is the cover story.

Step-Up Function Schema or "Buying Bags" Problem Type

Taught: Greg needs 16 balloons for his birthday party. Balloons come in bags of 10. How many bags does Greg need?

New cover story: Harriet serves ice pops to the 15 girls in her club. Ice pops are sold in bags of 3. How many bags does Harriet need?

Limiting instruction to problems with novel cover stories, which is what happens in schools, does not broaden schemas.

More Challenging, Schema-Broadening Transfer Features

- Different *Look*

- Different *Vocabulary*

- Additional *Question*

- *Irrelevant* Information

- Combinations of *Problem Types*

- Combinations of *Transfer Features*

Beyond The Cover Story

Taught: Greg needs 16 balloons for his birthday party. Balloons come in bags of 10. How many bags does Greg need?

Different Look:

The sign at the supermarket reads:

!!! Come and get it! Four frozen pizzas in

a bag for one low price!!!

You see the sign and decide to serve pizza for your next dinner party. You figure you'll need 10 pizzas. How many bags should you buy? (Select one answer.)

_____ 2   _____ 3   _____ 4   _____ 5

Beyond The Cover Story

- **Different Vocabulary**: Francis is buying eggs for her dinner party. She needs 26 eggs for the dishes she's making. Eggs are sold in <u>dozens</u>. How many dozens does Francis need?

- **Different Question**: Jose has $25 to spend on the ice hockey pucks he's buying for the team. He needs 7 pucks. Pucks are sold in bags of 3, and each bag costs $8. <u>After buying the pucks, how much money will Jose have</u>?

Our Instructional Framework

- Our MPS instructional framework relies on schema theory to explicitly teach students

  - To classify word problems into problem types

  - About transfer features beyond the cover story

- "Schema-Broadening Instruction" or SBI

Teachers and students refer to our schema-broadening instruction as

Hot Math OR

Pirate Math

A Look
at Pirate Math
First

Pirate Math Tutoring

48 sessions: 3 per week for 16 weeks

20-30 minutes per session

Scripted lessons, which tutors study (not read)

Four units

- Foundational Skills for Word Problems
- Total Word Problems
- Difference Word Problems
- Change Word Problems

Pirate Math: Introductory Unit

- Teach students:
  - Efficient counting strategies to answer math facts
  - Review of 2-digit procedural calculations
  - How to solve for X in addition and subtraction equations (a+b=c; d-e=f)
  - How to check work

Introductory Unit: Counting Up

COUNTING UP
Addition

1. **Put the bigger number in your head and say it.**

2. **Count up the smaller number on your fingers.**

3. **Your answer is the last number you say.**

To ADD, you CAN reverse the numbers!

Example: 5 + 2 = ?

1. *"5."*
2. (Hold up 1 finger.) *"6."*
   (Hold up 2 fingers.) *"7."*
3. *"The answer is 7."*

Example: 2 + 5 = ?

1. *"5."*
2. (Hold up 1 finger.) *"6."*
   (Hold up 2 fingers.) *"7."*
3. *"The answer is 7."*

COUNTING UP
Subtraction

1. **Put the minus number in your head and say it.**

2. **Count up on your fingers to the number you started with.**

3. **Your answer is the number of fingers you have up.**

To SUBTRACT, do NOT reverse the numbers. The minus number always goes first.

Example: 5 - 2 = ?

1. *"2."*
2. (Hold up 1 finger.) *"3."*
   (Hold up 2 fingers.) *"4."*
   (Hold up 3 fingers.) *"5."*
3. *"The answer is 3."*

Introductory Unit: Finding X in All 3 Positions of Equations

- X at the end (e.g., 3 + 2 = X; 6 – 2 = X)
    - Do what the problem says

- X not at the end
    - It's an "X minus" problem (e.g., X – 2 = 4)
        - Add
    - It's not an "X minus problem (e.g., X + 2 = 4; 6 + X = 10)
        - Subtract

Introductory Unit: Checking Work

CHECKING YOUR WORK

☑ 1. Sense

   Does the work make sense?

☑ 2. Lining Up

   Did I line up the numbers correctly?

☑ 3. Math

   Did I add or subtract correctly?

☑ 4. Labels

   Did I use a label?

☑ 5. Signs

   Did I use signs correctly?

Remaining Units:
Word-Problem Lessons

Following Unit 1, four activities per session.

1. Flash-card warm up

2. Conceptual/strategic lesson using schema-broadening instruction

3. Sorting practice on identifying problem types

4. Paper/pencil review

1. Math Fact Flash Card Warm Up

- Math Fact flash cards comprise 200 addition and subtraction facts
  - Sums 0-18
  - Subtrahends 0-18

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  11
-  6
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- Tutor shows flash card to student: Know it or Count Up!

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   4
+  5
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  - If student answers correctly, flash card placed in correct pile.
  - If student answers incorrectly, tutor asks student to "Count Up"; once correct, goes in correct pile.
  - Student graphs score on graph.

2. Math Fact Flash Card Warm Up

Math Flash 3 Warm-Up Flash Card Graph   **Student Name:**
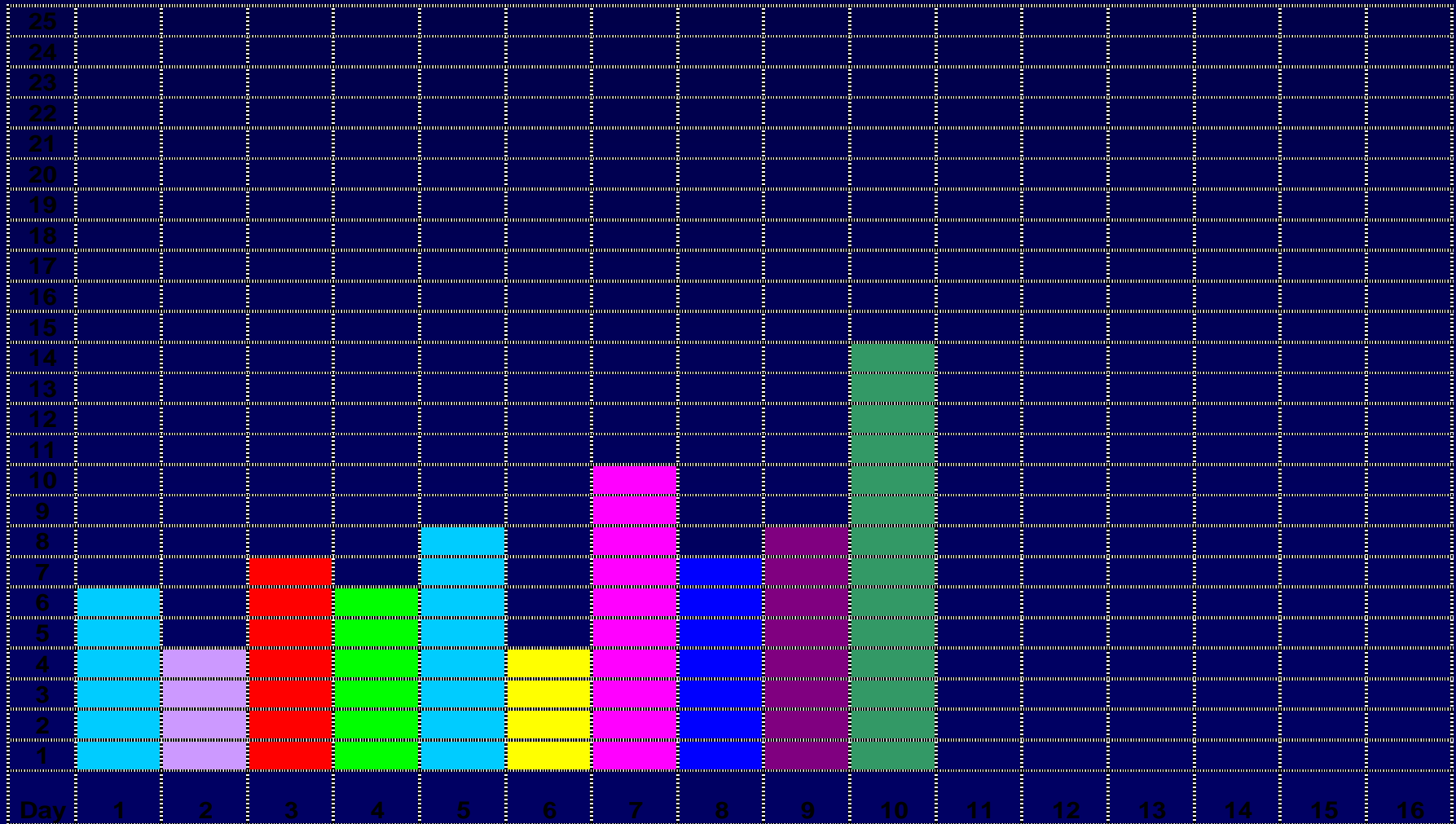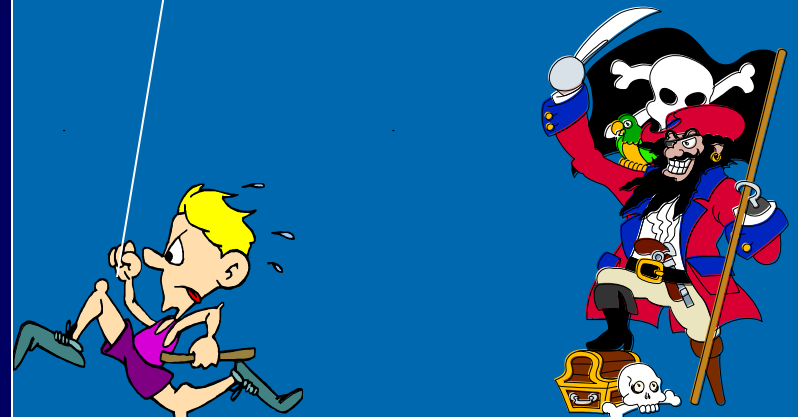
2. Lesson
Pirate Math RUN

- Students use "RUN" strategy for every word problem.

- Students circle relevant information and cross out irrelevant information directly in text or graphics.

RUN!

1. *Read* the problem.

2. *Underline* the question.

3. *Name* the problem type.

TOTAL, DIFFERENCE, OR CHANGE

At Grade 2

Three problem types (or schemas)

- Change problems
  - ➢ A starting amount that increases or decreases (a <u>change</u>) to make it a new amount

- Total problems
  - ➢ Two parts are combined to make a a <u>total</u>

- Difference problems
  - ➢ Two amounts are compared to find the <u>difference</u> between them

2. Lesson
Pirate Math Setting Up Work

- Write the "meta-equation" equation that goes with the problem type.

- Figure out what's missing. Write X in appropriate slot of equation.

- Figure out what numbers are given. Write those numbers in appropriate slots.

- Write math signs.

- Find X.

- Make sure answer has a number and a label.

2. Lesson: Transfer Instruction

- Transfer features:
    - Irrelevant information
    - Finding relevant information in graphics
    - Combining problem types

- Explicit instruction on how to transfer: Teach transfer features in connection with that unit's problem type so students recognize novel problems as belonging to a problem type for which they know the solution method

2. Lesson
Pirate Math Change

- Change problems with a starting amount that increases or decreases (a <u>change</u>) to make it a new amount.

- "Sarah had 10 pencils. Then she gave 4 pencils to Pamela. How many pencils does Sarah have now?"

- St +/- C = E

"Sarah had 10 pencils. Then, she gave 4 pencils to Pamela. How many pencils does Sarah have now?"

"Sarah had 10 pencils. Then, she gave 4 pencils to Pamela. How many pencils does Sarah have now?"

Name problem type: Change problem

Write meta-equation: St +/- C = E

Identify missing information: St +/- C = X

Enter givens: 10 +/-   4   X

Write math signs: 10 -   4 = X

Find X: X=6.

Label answer: 6 pencils

Lexie had some comic books in her desk. Then she bought 8 more. Now, she has 12 comic books. How many comic books did Lexie have in her desk to begin with?

Lexie had some comic books in her desk. Then she bought 8 more. Now, she has 12 comic books. How many comic books did Lexie have in her desk to begin with?

Name problem type: Change problem

Write meta-equation equation: St +/- C = E.
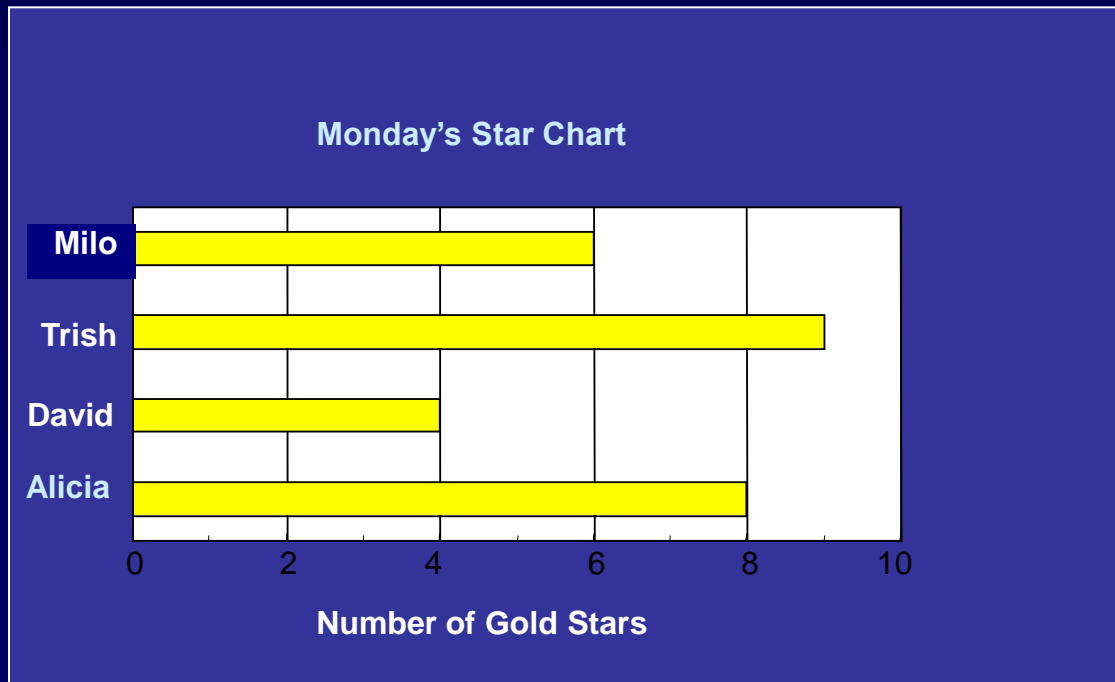
Identify missing information: X +/- C = E

Enter givens: X +/- 8 = 12

Write math signs: X +   8 = 12

Find X: X=4.

Label answer: 4 comic books

Alicia has 3 friends in her math class. The chart shows how many stars Alicia and her friends earned on Monday. On Tuesday, Alicia lost 3 stars for talking. How many stars does she have now?

Monday's Star Chart

	Number of Gold Stars
Milo	6
Trish	9
David	4
Alicia	8

Alicia Solution

- Transfer features:
  - irrelevant information
  - finding relevant information in graph

- Change problem

- St +/- C = X

- 8 +/- 3 = X

- 8   -   3 = X

- X = 5

- 5 stars now

2. Lesson
Pirate Math Total

- Total problems have two parts that are combined for a <u>total</u>.

- Total amount is the entire or combined amount.

- "Sarah has 5 pencils. Pamela has 3 pencils. How many pencils do the girls have in all?"

- P1 + P2 = T

2. Lesson
Pirate Math Difference

- Difference problems compare two amounts to find the <u>difference</u> between them.

- "Sarah has 7 pencils. Pamela has 12 pencils. How many more pencils does Pamela have than Sarah?"

- B – s = D

3. Sorting

- Student sorts word problems by problem type for 2 minutes.

- Tutor reads cards to student.

- Student places cards on Sorting Mat.

- At end of 2 minutes, tutor counts number of correctly sorted cards and uses correction procedure for up to 3 incorrectly sorted cards.

3. Sorting

Sorting Mat	
Total	**Difference**
Change	**?**

3. Sorting

Maria and Jackie picked 16 flowers. Jackie picked 7 flowers. How many flowers did Maria pick?

Maria picked 8 more flowers than Jackie. Jackie picked 4 flowers. How many flowers did Maria pick?

Maria picked 11 flowers. Then Jackie took 4 of them for her Mom. How many flowers does Maria have now?

3. Sorting

Jeremiah saw 27 sharks and 19 turtles at the aquarium. How many more sharks did Jeremiah see?

Jeremiah saw 33 sharks at the aquarium. He saw 14 turtles. How many sharks and turtles did Jeremiah see?

Jeremiah saw 29 sharks at the aquarium. Then 12 of the sharks swam away. How many sharks could Jeremiah still see?
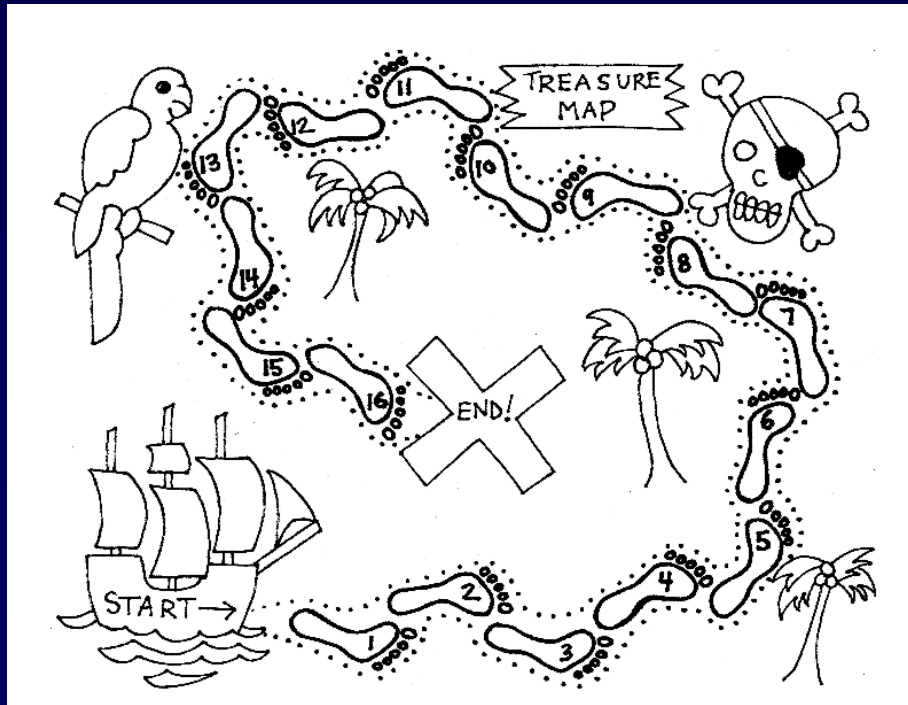
4. Paper/pencil review

* 10 math facts
* 4 double-digit calculations
* 1 word problem

Motivation during *Pirate Math*

- Students earn coins throughout lesson for listening well, working hard, following directions, and correct work.

- At end of lesson, students color footsteps on treasure map equaling amount of coins earned.

- When students color 16 footsteps, they pick a prize from treasure box and receive a new map.

Based on A Series of Field-Based Randomized Control Trials at 2nd and 3rd grades

Lynn Fuchs, Sarah Powell, Pamela Seethaler, Paul Cirino, Jack Fletcher, Doug Fuchs, and Carol Hamlett

Vanderbilt University and University of Houston

Participants

- Screened 924 students in 63 classrooms in 18 schools in two cities

- 133 met inclusion criteria:
  - Low on math calculations
  - Low on math word problems
  - Not low on IQ

- Students randomly assigned to tutoring conditions:
  - Math Facts Tutoring
  - Word-Problem Tutoring (Pirate Math)
  - Control

Examined Effectiveness
of Two Tutoring Programs

Both Tutoring Programs

- Delivered individually
- 48 sessions: 3 per week for 16 weeks
- 20-30 minutes per session
- Scripted lessons, which tutors studied (not read)
- Motivational system to ensure on-task behavior and hard, accurate work
- Each session audiotaped; tapes sampled and coded for fidelity, which was high for both tutoring conditions

Examined Efficacy
of Two Tutoring Programs

- The exclusive focus of math facts tutoring was <u>math facts</u>

- The primary focus of Pirate Math tutoring was <u>word problems</u>
  - but it also addressed foundational skills (math facts, procedural calculations, and algebra)

Outcomes

- Math Facts
  - <u>PM</u> = MF > no tutoring

- Procedural Calculations
  - <u>PM</u> > MF = no tutoring

- Word Problems
  - <u>PM</u> > MF = no tutoring

- Algebra
  - <u>PM</u> > MF = no tutoring

Algebra

- Algebraic cognition improved even though students were severely deficient in math and young.

- Given strong focus on algebra in high schools, given graduation requirements for algebra, and given emphasis in NMAP, introducing algebra earlier in the curriculum may represent a productive innovation.

Correct Representation

Incorrect Representation
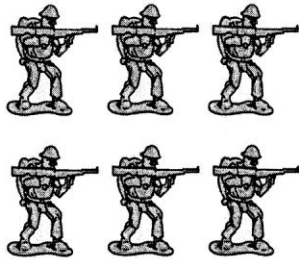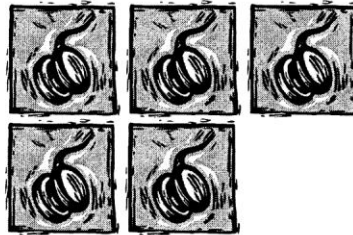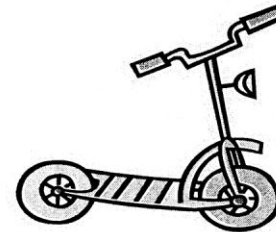
These Tutoring Protocols Are Transportable

Tutoring protocols were comparably effective in Nashville (where tutoring program was developed) and Houston (a distal site).

Conclusions

For a reasonable amount of tutoring time (48 sessions, each 30 minutes long), Pirate Math enhances word-problem skill, fluency with math facts, procedural calculation skill, algebraic cognition, and competence with word problems.

With a 1-day training, ongoing supervision, and tutors studying scripts before delivering them, non-certified tutors can implement Pirate Math at distal sites with comparable outcomes.

What About Alignment Between

Primary and Secondary Prevention?

A Study to Examine Alignment Question

- RTI integrates assessment and intervention within a multi-level prevention system to identify and reduce risk for academic failure.
  - All children receive primary prevention (core instructional program).
  - Students at risk for poor outcomes receive secondary prevention (validated small-group tutoring).
  - Students who do not respond to secondary prevention proceed to tertiary prevention (more intensive tutoring individualized to meet a student's nonstandard needs).

Purpose of Study

- Examine mathematical problem-solving learning and explore prevalence of MD as a function of
  - Hot Math classroom prevention
  - Hot Math tutoring
  - Whether Hot Math tutoring occurs with or without Hot Math classroom prevention
  - Focus on AR and NAR learners

Effects of Small-Group Tutoring with and without Validated Classroom Instruction: Are Two Tiers of Prevention Better Than One?

Lynn Fuchs, Doug Fuchs,

Caitlin Craddock, Kurstin Hollenbeck, Carol Hamlett

Vanderbilt University

Chris Schatschneider

Florida State University

Importance of Study

- Simultaneously evaluating the importance of primary and secondary prevention for designing efficient and effective RTI prevention systems.

- If tutoring is differentially strong when combined with validated classroom instruction, then both levels are critical, and tutoring should be provided as a supplement to classroom instruction.

- If tutoring promotes comparable outcomes regardless of the classroom instructional context, then tutoring might occur as a replacement for classroom instruction. This would make RTI more feasible and efficient.

Figure 1

Example of Each of Four Problem Types for Which Solution Rules Are Taught

Shopping List Problem Type

Danny needs to buy things for his science project. He needs 2 batteries, 3 wires, and 4 magnets. The batteries cost $3 each, the wires cost $3 each, and the magnets cost $2 each. How much money does Danny need for his science project?

Half Problem Type

Dave and Todd are going to buy a large box of baseball cards. There are 42 cards in the box. Dave and Todd will each get 1/2 of the cards. How many cards will each of them get?

Bag Problem Type

You want to buy some lemon drops. Lemon drops come in bags with 10 lemon drops in each bag. How many bags of lemon drops should you buy to get 32 lemon drops?
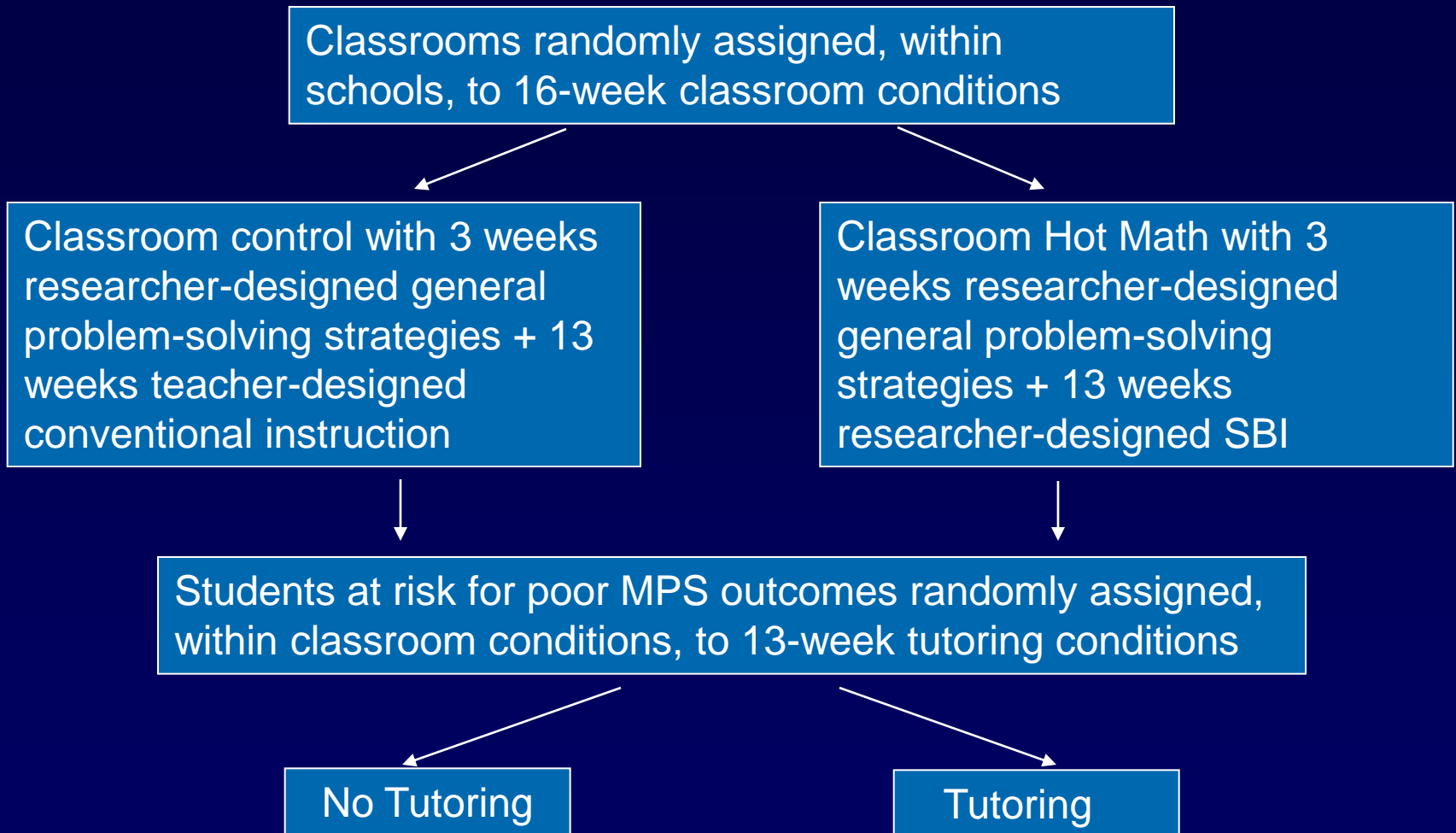
Pictograph Problem Type

Gloria collects teddy bears. She made a chart to show how many teddy bears she had. Each picture of a bear stands for 4 bears.
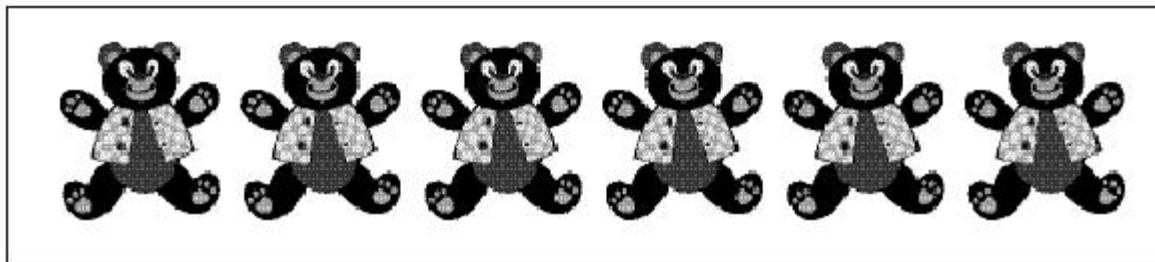
For her birthday, Gloria got 3 more teddy bears. How many bears does she have now?

Study Design

Classrooms randomly assigned, within schools, to 16-week classroom conditions

Classroom control with 3 weeks researcher-designed general problem-solving strategies + 13 weeks teacher-designed conventional instruction

Classroom Hot Math with 3 weeks researcher-designed general problem-solving strategies + 13 weeks researcher-designed SBI

Students at risk for poor MPS outcomes randomly assigned, within classroom conditions, to 13-week tutoring conditions

No Tutoring

Tutoring

Note: Two-thirds classrooms in classroom Hot Math. Two-thirds AR students in Hot Math tutoring.

Study Conditions

- FOR NAR
  - Control (teacher-designed word-problem instruction)
  - Hot Math Primary Prevention

- FOR AR
  - No Hot Math (teacher-designed word-problem instruction)
  - Hot Math at Primary Prevention
  - Hot Math at Secondary Prevention
  - Hot Math at Primary + Secondary

All conditions addressed same 4 problem types and had comparable instructional time.

Hot Math

- Primary Prevention
  - First unit on general math problem-solving strategies (control group also received this)
  - Four additional Hot Math units, each dedicated to one problem type
    - Explicit instruction on <u>skill acquisition</u> (only cover stories vary; no problem used for instruction used for pre- or posttesting)
    - Explicit instruction on how to <u>transfer</u>: Teach transfer features in connection with that unit's problem type so students recognize novel problems as belonging to a problem type for which they know the solution method (no problem or specific transfer challenges used for instruction used for pre- or posttesting)

Hot Math

- Secondary Prevention
  - Is more intensive.
  - Occurs in small groups (2-4).
  - Focuses on difficult concepts within Hot Math.
  - Employs a systematic reinforcement system to encourage on-task behavior and hard, correct work.

Sample

- Conducted in 4 cohorts, one cohort per year for 4 years
- 17 schools
- 119 3rd-grade classrooms (1 Hot Math teacher left study in first month for personal reasons)
- Within schools, classrooms randomly assigned to conventional vs. Hot Math
- Screened 2,023 students on whom we had consent
- Randomly sampled 1,200 students for participation, blocking within classroom and within 3 strata on Test of Computational Fluency:
    - 25%: 1 *SD* < mean of entire distribution
    - 50%: 1 *SD* within mean of entire distribution
    - 25%: 1 *SD* > mean of entire distribution.

Sample

- 2-subtest WASI IQ: 97.29 ($SD = 13.93$).

- WJ III Applied Problems: 102.56 ($SD = 13.60$)

- WRMT-R Word Identification: 100.83 ($SD = 10.09$)

- 49.0% male

- 54.9% received subsidized lunch

- 42.1% African American; 40.7% European American; 10.5% Hispanic; 1.5% Kurdish; 5.3% other

At-Risk Students

- 288 students identified as AR for poor outcomes

- Within each cohort, rank ordered students on predicted score and selected the lowest 72 students in that year's sample as AR.

- Within cohort, AR students assigned to Hot Math tutoring or no tutoring, while stratifying by classroom condition.

Final Sample, After Attrition

- 119 3rd-grade classes
  - 30 conventional classroom instruction
  - 89 Hot Math

- 898 NAR students
  - 298 conventional classroom instruction
  - 600 Hot Math

- 243 AR students
  - 79 no tutoring
  - 164 Hot Math tutoring

Comparability

Teachers
> **Demographics**
> **Minutes of math instruction**

NAR Students
> **Demographics**
> **Reading/math/cognitive performance (but higher than AR students)**

AR Students
> **Demographics**
> **Reading/math/cognitive performance (as function of classroom and tutoring conditions)**

Study Information

- Students pre/posttested on 4 MPS measures

- Classroom Hot Math implemented for 16 weeks

- Tutoring Hot Math implemented for 13 weeks (began in Week 4 of Classroom Hot Math)

MPS Measures: All Novel Problems
How Varied from Problems Used for Teaching Problem Solutions
(where cover stories were the only source of novelty)

- Immediate Transfer
  - Structured analogously (new cover stories)

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  - Novel look, novel question, novel vocabulary, irrelevant information, combining 2 problem types and 2 transfer features

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- Near Transfer
  - Novel look, novel question, novel vocabulary, irrelevant information, combining 2 problem types and 2 transfer features
- Far Transfer
  - Formatted to resemble commercial test
  - Multi-paragraph narrative
  - Some relevant information removed from narrative (some the student has to determine)
  - Multiple pieces of narrative and numerical irrelevant information
  - Combined all 4 problem types (plus other problem types) and varied all transfer features

Shopping for School Clothes

School is about to start, and it's time to shop for school clothes. Your mom makes a list of the clothes you must buy, and she asks you to think of other things you want for school. You and your mom decide that you'll make two shopping trips.

On the first trip, you'll buy the clothes she says you need. Your mom says you must buy four pairs of jeans, six shirts, ten pairs of socks, two pairs of shorts, and one pair of shoes. On the second trip, you'll buy the things you want.

Your mom has saved one hundred five dollars to spend on your clothes. You earned ninety-seven dollars for clothes by washing twenty-three cars.

Clothing Prices

1 Pair of Jeans. $17

1 Shirt . . . .$ 7

1 Pair of Shorts. . . . . $ 6

1 Pack of Socks
   (4 pairs in a pack). . . . . $ 8

Regular Shoe Prices (See Key)

Nike	👟👟👟👟👟👟👟👟
Adidas	👟👟👟👟👟
Converse	👟👟👟👟
Reebok	👟👟👟👟👟👟👟

Each 👟 means $10.
All shoes are on sale for $\frac{1}{2}$ the price on the chart.

1. How much money do you have for school clothes?

2. How much money will you spend on your first shopping trip on jeans, shirts, socks, shorts, and shoes? Show all your work.

3. On your second shopping trip, you can buy some other things for school. What will you buy and how much will you spend? What money could you use to pay for these things? (For example, how many $1 bills, how many $5 bills, how many $10 bills.)

4. School sweatshirts cost $12. After your shopping trips, will you have enough money to buy one? Explain how you got your answer.

MPS Measures: All Novel Problems
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  - Some relevant information removed from narrative (some the student has to determine)
  - Multiple pieces of narrative and numerical irrelevant information
  - Combined all 4 problem types (plus other problem types) and varied all transfer features
- Math Applications (WJ III-Applied Problems)
  - Counting, telling time or temperature, problem solving

Findings

- Across NAR and AR students:
  - Classroom Hot Math was more effective than conventional classroom instruction (effect sizes > 1 standard deviation).

- For AR students:
  - Hot Math tutoring was more effective than no tutoring (effect sizes > 1 standard deviation).

For AR

Secondary + primary vs. secondary: ES=1.34

So two levels of prevention are better than one.

Findings: MD Among AR

- Hot Math Tutoring, but not Classroom Hot Math, reduced MD prevalence (< 16th percentile on WJ III Applied Problems at end of study).

  - Conventional class without tutoring: 28.5%
  - Hot Math class without tutoring: 25.5%
  - Conventional class with tutoring: 12.5%
  - Hot Math class with tutoring: 12.9%

Results Indicate …

- Need for validated MPS instruction at primary prevention for NAR and AR students.

- For AR students, two levels of validated prevention are better than one.

- In this study, validated classroom and tutoring were closely aligned theoretically and operationally. If not aligned, results might differ. So, we need to be thoughtful, with flexible interpretation of "participation in the general education curriculum."

For information on how to obtain
Pirate Math or Hot Math manuals,

flora.murray@vanderbilt.edu.