

DAVID CHARD: Let me begin by thanking what Jackie referred to as PaTTAN, which in Texas I refer to as PaTTAN because – oh, good, so it's just a different – This is, I think, maybe the fourth or fifth time I've presented for this group, and I've really enjoyed my opportunities to come to Pennsylvania and I've come back.

A little bit about what we're going to do today. I'm going to spend the first 40 minutes or so talking about issues related to mathematics instruction as we think about growing children who are confident and competent in their understanding of mathematics, things we should think about, both as society as well as parents and teachers. Then we'll take a break at 2:45 so you can grab some refreshments and water. We'll come back at 3:00 and I'll spend another about 40 minutes or so talking about a very specific study that we've been conducting on the west coast and in the southwest, focusing on kindergarten mathematics development that is a federally supported grant. And I want to show you some of the things we're learning as a result of that study, more than just kind of the horse race of does one program do better than another. So, that's sort of what I'd like to talk about.

So, what makes me any authority on the topic of mathematics education? I was a high school math teacher. I taught in Southern California, suburban Detroit, and I taught in the U.S. Peace Corps in mathematics and chemistry, and so I have kind of taught a full range of children at the middle and high school level, and I've also turned my attention to developing programs, if you will, curriculum programs for children at the elementary grades, in particular, thinking about students that we ultimately often identify as having special needs as a result of their failure to be strong in mathematics content. So, with that, I'm going to talk a little bit about kind of an emerging understanding about math education as it relates to our research around struggling learners.

But before I get there, I had a really interesting opportunity last night. I'm on the board of an institute focused on engineering education in the United States. Particularly focused on trying to improve higher education opportunities for women and children of color to become engineers. Right now the M.I.T. is probably doing the best job of getting gender parity in engineering education, with about 43 percent women and the balance being young men. But the bigger concern we have is that we're not really tapping the resource of the children in our country to become engineers. And over the past several years what we've noticed is a steady decline in the number of individuals seeking undergraduate degrees in engineering. And the per capita engineering degree. So, if you have a population that's grown. The absolute number has sort of continued to fall slightly, but the per capita number with a growing population is stunningly low, and when you compare it to other countries, China, for example, has an exponential growth in per capita engineers. So, they're looking at the future and saying if we don't grow individuals who can answer our problems, who can take

on scientific and mathematical challenges, we're going to be in deep trouble as a society. And we as a country are the ones facing the deep trouble.

So, the other members of this board that I sit on were the Secretary of the Navy and the Secre – we have defense personnel, we have the gentleman who invented the noise-cancelling headphones for Bose – a very interesting set of people who have a very serious concern about our ability to supply children from K-12 or pre-K-12 education who are ready to pursue science and mathematics degrees.

So, if you ratchet that back and say where does the problem begin, what we have seen with our international data on children's development in mathematics, the U.S. has about 8 percent of the children who take the TENS exam who actually reach advanced levels compared to other countries that are in the 26, closer to 30 percent. Singapore's in the 40s. Alright.

So, then the question is why? Why is this happening? And I believe it's because we've not done a good job of addressing what is a really detailed problem. This is not like reading. So, one of the challenges I think we have is recognizing that the approach to improving math education will not follow the same trajectory as the approach to improving reading education. It will not. And I will get to, in my talk, why that's the case. But, our traditional ways of dealing with challenges in our achievement will not work in this case.

Secondly, this is not a new crisis; right? Suddenly people are paying attention outside of our field, but I started teaching in 1984 and I was one of about 13 people that the State of California bribed to come and teach mathematics. They could not hire math teachers. We just didn't exist. Schools were not producing them. And that problem still exists today. Getting people in mathematics and engineering and science fields to pay attention to the opportunity to teach, and you all know why, right? It's a tough job. We don't get compensated fairly. And, we're expected to stay at a field for a long time, most of which young children today, young people today, don't do in any job. So, we've got a lot of systemic factors going on.

And, by the way, I was a high school teacher. I was sharing with Jackie earlier, so jump in if you have questions, Alright. I would prefer not to be standing on the stage and better if I could be close by, but if you have questions, ask them; don't wait.

So, how do we get in a better place? How do we improve the situation and find ourselves in a place of preparing children to be competent and confident in mathematics, and science for that matter, and increase the number of people who are interested in focusing in things like the field of engineering. Because if you can be an engineer, you can do almost anything, literally, as a career. Right? There are tons of opportunities.

So, we have begun to think about conceptual framework for thinking about the problem, and it's bookended, of course, by having a set of shared expectations, which we're growing closer to with the common core standards that the current presidential administration is supporting. But currently every state has, with the exception of Iowa, has a set of standards. Those standards by and large in mathematics do not differ from state to state. It doesn't matter whether you're living in South Texas or the State of Washington, or here in Pennsylvania, the standards are relatively similar about the content. Why is that? This is not a rhetorical – I tend not to ask rhetorical questions, so why is it that the standards from state to state are relatively similar? Mathematics is mathematics and it is not new; right? We've been doing it for centuries. So, the content itself is related – the problems we apply it to are very different, right, than what we were applying it to centuries ago, but mathematics is a universal language that has not changed significantly in a very long time.

So, standards and expectations. On the other end we are now a system of education that is being held accountable for our children's outcomes and we applaud that because any good profession expects to be held accountable for what it does, so we like that. We have some work to do in the accountability world. We all know that. But, nevertheless, it's a good thing.

So, in the middle we are proposing, we really need to work on some specific details. My apologies. If you can't see or if I get in your way, just run over. So, one is the physical environment, which I won't talk about a lot today, but there are lots of things to think about in terms of the efficiency model of education that we currently run; right? Thirty plus kids to a classroom with one teacher. Probably not the best model for developing high class outcomes. I mean, if we re-thought that and really had all the resources we could work with, 30 to one probably wouldn't be the way we'd go about it. Would you agree? Okay, it's late in the day. You have to humor me, right? You will have to nod periodically and make me feel like I am talking to you.

And, we have to think about instructional tools and materials. What do I work with here? What are the best – and I will touch on these from time to time and I'll spend the second part of my presentation talking about a specific tool that we're developing and testing for teachers to use. But I really want to focus most of my attention on the middle. The teacher knowledge and practice. And this really is beginning to shape up with 2 specific research efforts into 2 piles about what teachers need to know. One pile is how much math do I need to have? If I'm a second grade teacher, what is the level? Do I need calculus? Or do I need algebra? What do I need to know? And on the other pile, what do I need to know to do with it; right? How do I break that down? How do I sequence instruction? How do I move from simple to complex? How do I make this concrete for some children and abstract for others? How do I move them along the continuum of getting increasingly mathematical in their thinking and less

concrete?, because that's the idea. If we leave children in pre-K-12 education only operating at concrete level, we've failed. Alright. We have to get them to an abstract level where they really think mathematically in a very abstract way.

So, it's that teacher knowledge and practice piece that Debra Ball and Heather Hill started at the University of Michigan and have begun to really focus on. Can you assess that in teacher's thinking? And is there sort of a benchmark level that we should expect a teacher to have to teach, let's say kindergarten through second grade? Is there a different benchmark for teachers to have from third grade to eighth grade? Just what level of knowledge do I need to be a really good teacher of that particular content?

And then the other piece is a recent study published in the *American Educational Research Journal* that showed that these two, they kind of confirmed Ball and Hill's findings that these two things, these two types of knowledge really do exist, are testable. They are related to children's outcomes. And they've done it in multiple countries. Germany and the United States. So, we're beginning to see, again, more emerging knowledge about this.

Questions about this sort of powerful core variables that stand between children's – our expectations and their achievement?

AUDIENCE MEMBER

DAVID CHARD: One of the best articles to read was in – on the website. So, I like things that are accessible, right, so on the website for the American Federation of Teachers, you can download their journal. Whether you're a member of the union or not, you can join that, you can download their journal, which is called *The American Educator*, and I don't think it's searchable. You might have to look at each volume, but there are a number of issues that focus on mathematics education, and one of them includes a number of articles by Debra Ball and Heather Hill.

How many of you are school district leadership or school leadership? If you're in a leadership position, you should probably know that Heather Hill, who is now at Harvard, will give you the test and all of the materials and training free of charge. All you have to do is get to one of their trainings, wherever it happens to be. And the beauty of it is it all comes on a CD and you can modify the assessment in different ways. And it really could be sort of the basis of a study group for teachers. What do I know here? Rather than just an assessment, it could be used in another way to stimulate conversation, where do we need more support, etc. So, Debra Ball, Heather Hill.

Other questions? Okay. So, let me try to organize this in a way that will kind of help us guide our conversation. And I really would like you to jump in if you have questions.

So, thinking from a teacher perspective, I sort of try to understand what variables I have to control. Right? We're teachers, we're all sort of control freaks at some level. We like to know what we can fix and change and do, and I'm accused of being a problem solver and sometimes to my own peril. But I like to think about what mathematics do I need to know to teach? What are the fundamental pieces of mathematics I need to understand in order to be effective at teaching? Who are my students and what do they need to know? And then lastly, what tools and practices and programs are available for me to accomplish my goal effectively? And I want to begin with these two pieces. This fundamental mathematics for teaching and fundamental kind of powerful understanding of mathematics and really sort of dig in there.

So, one of the problems with mathematics, I believe, is the issue about it being such a historical topic. The connections to faith, religion, why it was avoided for centuries, no one would even talk about it. Where it eventually came from and why we use a circle symbol to represent zero, where that started. So, there are books that really bring mathematics alive that I think we need to introduce children to; and we don't. These things tend to be very separated. Some of the most fascinating human beings in history were brilliant mathematicians and they got into things that would get them on TMZ today; right? Our brawls, suicides, homicides, you name it. These people were not what you'd call your average everyday person. They were a little out there, right? Which is why we benefited from their knowledge. But if we made those people real for kids, they would begin to understand that mathematics is - was a language that was developed for solving problems. That's what it is. You've got a problem, you figure out how to solve it, and the best way to do that is mathematically, right? So, any physical problem that we need to model happens to be best modeled through mathematics.

So, think about, first of all, making yourself more knowledgeable about the history of mathematics. Where did this come from? Who are the prominent people involved? How did they - I mean, there are people in our society who spend all of their time trying to solve one mathematical equation, and they get paid lots of money to do it, because it has national security connections. I mean, that's what they do. So, we need to make sure our students know this. Get them excited about this topic that typically we don't. We don't get them excited.

Secondly, we need to really take on the challenge of being better educated about mathematics ourselves. And this one is one of my personal issues. I hate when - so don't do this. I hate when people say, "I'm not a math person." Yes, you are, as a matter of fact. Otherwise you wouldn't be as successful as you are. You use mathematics every day. It doesn't mean you are a mathematician, but you are a math person. And we need to take on this just like we do with the, you know, Dr. Seuss Reading Across America Day. We need to make this a serious cultural change in our schools. Otherwise, that

growing number of people who don't go into science and mathematics will continue to grow. We have to get children excited about mathematics, and the only way to get there is by you being excited and getting parents excited. And the best way to get there is to get knowledgeable, right? Understand this. Take on that challenge. Yes?

AUDIENCE MEMBER

DAVID CHARD: At eighth grade.

AUDIENCE MEMBER

DAVID CHARD: Absolutely. I'll come back to this, but this in and of itself is interesting. Look at the standards for children in fourth grade in Hungary. The mathematics standards for children in fourth grade in Hungary are about the equivalent of the mathematics standards for U.S. kids in the eighth grade. We just have low expectations. And guess what? We need them; right? So if one of the things we have to get serious about is increase our expectations. We need to begin teaching algebra much earlier. It's not a distinct topic. It is an extension of arithmetic. We can teach it as early as pre-K kindergarten. And build kids' understanding of it along with their understanding of arithmetic. But that's not what we do. We kind of encapsulate into a course. And we've only run further down that road rather than kind of pulling back and saying how do we increase our expectations for children earlier on.

So, get knowledgeable about number systems, number relationships. I'm just going to try to give you a feel for what I mean by that and then I have to leave you to question just what do I know? And what would be a really enjoyable way - I'll come back to that - of increasing my knowledge. Because if I give you a punishing thing to do like getting smarter about mathematics, the likelihood that any of you are going to do it is quite small. I want you to do it, so I want you to think about how could this be made fun for teachers?

So, let's talk about number systems. Because those of us who were kind of raised in a new math world, right, were introduced to number systems, and then new math was thrown out and we went back to basics because of a nation at risk and we threw out a lot of the good stuff as well as the stuff that maybe wasn't as beneficial for all kids. But one thing we need to learn and introduce kids to is that there's a set of number systems that define number operations. So, what I mean by that is we have counting numbers, or natural numbers, and whole numbers once we've included zero, and then integers when we begin to talk about negative numbers, and it's only there that we can begin to talk about subtraction. Right? So, without understanding the system of integers, we can't really teach children fully about subtraction, and yet, we separate these two things by years. Right? We teach kids to subtract much earlier than we introduce integers to them. Probably not necessary. Probably need to rethink

that. But the standards also reflect that. So, I'm really talking about a significant revolutionary change if we're going to make something happen.

Then, it's only when we introduce rational numbers that we begin to, begin to, begin to be able to define division. Because without the concept of rational numbers, right, we can't conceptually – we can talk about division, but we can't abstractly talk about division and the properties that define division.

Yes?

AUDIENCE MEMBER

DAVID CHARD: They may not introduce the word, but they introduce the idea, absolutely. So, imagine the difference, though. What do we do mostly when we talk about subtraction? We talk about takeaway models. Right? That's what we typically do. So, if you talk about difference models, not just takeaway models, what are the difference between two numbers, you begin to show that we're talking about difference in magnitudes. And then from there we can easily move to numbers that are different from zero, that have a lower magnitude than zero. It's not that complex in an abstract way for young children to get it. And then, of course, you begin to talk about other kinds of numbers and the real numbers that all of these things fall under. Not the imaginary ones, which they'll learn later on.

But, the point is, is that you have to ask yourself, do I know enough about this in order to teach this in a comprehensive way so children begin to understand that there are systems and the systems behave differently? Because what we tend to do is a very good job, for the most part, of up to here. Mastery of whole number concepts we do a pretty good job of as a system. We do a fairly good job of integers, and we fall completely apart here, making the bridge to rational numbers. We really struggle. And we struggle in large part because many teachers don't understand it. And it's complex, don't misunderstand me. Try to explain to a friend why division of fractions works. Very hard to do, and it's very hard to show. So, we'll talk a little bit about that as well.

After the break, I'm going to tell you about how we're beginning to talk about the earliest instruction in kindergarten and trying to show effects that provide better instruction for low achievers. Okay, that was our goal with what we call the ELM Study. The conceptual framework here being that we wanted to work really hard on developing mathematical concepts and models very early with children and that these were going to be deliberate and they were going to be few. We weren't going to try to cover the typical kindergarten standards. Rather, we're going to teach few things really deeply. So, I'll tell you a little bit about that.

Then we went at the issue of mathematics related vocabulary and discourse. Believing that this is a very ripe time during language development. That if we could get kids talking mathematically at the age of five, we might be able to see a transfer of that effect with children as they grew up.

And then we also attended to the issue of procedural fluency. Because one of the things that we have, as a country, I believe, one of the hurdles we've gotten to avert, is that we no longer think about conceptual understanding and procedural fluency as two distinctly different ideas. They feed one another and our cognitive development is built on both. So, if you're a middle – are there any middle school teachers? Okay. A few middle school teachers. Often middle school teachers will say to me, "If only these kids knew their math stats." Well, they're right. If you don't know your math stats, it's much more challenging to do complex mathematics. In part because you don't know your math facts because you don't understand the concepts. So the two work together. So we built into the kindergarten program that we're building, procedural fluency as well. So, I'll tell you more about that as we go through our presentation.

The next piece is understanding, in understanding these mathematics fundamentals is understanding mathematical principles. Right? So, you remember all of them. You teach them. The commutative property, identity element, all of these things. These things are absolutely precious pieces of information that kids have to understand are laws. They are not just ideas. They are laws that cannot be broken. Right? The beauty of mathematics is that it is a lawful language. You can't change some of these early fundamental ideas. Commutativity of addition, commutativity of multiplication. Absolutely lawful. And it always works. Therefore, it works even when we don't know what the numbers are. Right? That's where the move to algebra becomes difficult. If I say to a child, 2 plus 3 is the same as 3 plus 2, and they study it for a few minutes, and maybe model it with some concrete tools, they'll get that. What we really want them to understand is that $A + B$ is the same as $B + A$ because it's a law that can't be changed. Does that make sense? Okay. So that lawful idea is really, really important.

And then coming up with appropriate mathematical models is also absolutely essential. Let me give you kind of an important example here. So, I watch kindergarten and first grade teachers teach patterning a lot. Alright. And they have been told through professional development and their own insight that patterning is important for later insights into algebra. Absolutely true. But random patterns, not so important. What we want children to understand is that ultimately, is that patterns need to be related to functions so that when they see the X, if you will, of the function, they can predict the Y of the function. So, patterning, in and of itself, not so important. Patterning that develops functional understanding of functions, extremely important.

So, one of the things I often do is look at the way we structure textbooks or program materials, tools that teachers use, and whether they encourage this gradual movement to the understanding of functions in the early grades. Or whether they keep it random. It always starts random, don't get me wrong, because we want kids to feel comfortable with the tools, etc., but we need to nudge them to thinking about functional understandings.

We also want them to understand that they can use mathematical tools of any sort. So, these are models that we've built into the ELM Program. Nothing will be particularly revolutionary to you. Number line. Hundreds chart. Finger representations. Talley marks. Fighting ten sprains. And naive teachers often use one model too much. The key here is that we want them to see that mathematics is lawful regardless of what the concrete is. As long as you got enough of them. Fingers are a bit limiting. Alright. But, you've got to move toward using lots of different models to demonstrate the same idea.

In July – I'm working on an international project with World Bank in Kenya. Kenya is one of the countries in Africa that has recently adopted universal primary education, which is fantastic because now kids don't have to pay to go to school and everybody gets to go to school at the same time, etc. And they have standards. They've had standards and accountability systems for decades. But one thing they don't have necessarily is a lot of well-educated teachers. And so we were meeting with a group of teachers developing assessment to use in the primary education in Kenya and we were talking about the use of the number line. And the teacher said, "We don't teach the number line until grade eight." This is a significant problem. The number line should be taught, if you can, in kindergarten, because it is absolutely the most powerful arithmetic model for kids in the early grades. It can support their development of all kinds of understanding of operations, and it helps them move from the additive concepts to multiplicative concepts. It also helps them move from those number systems we talked about, moving from the whole numbers to rational numbers. We didn't convince them to change it, but they need to seriously consider just how important it is to keep tradition versus to advance their children's education.

When we model fractions, which is that important transition to rational numbers, we have to think about a wide range of models and not stick to pizzas, because that tends to be where we land. Why? Because it's familiar. But we have to make sure kids understand that there are set models of fractions, and that there are area models of fractions. And then we need to show them number line fraction representations because it will help them fraction equivalence.

As you can see down on this bottom – I really hate standing up here. I'd be over there, if I could, but you can see down here this fraction equivalence. We want them to begin to see fraction equivalence on the number line, which is a concrete for them because they've used it all through their whole number development. Right? Very familiar. They begin to see that one-half is the same

as two-fourths, and they can see where the denominator and numerator exist there and make sense of that very quickly.

The next area that we need to think about is understanding that common algorithms are derived from formulas. This is one that I continue to struggle with.

Excuse me for just a moment. My daughter says I have a high metabolism. I sweat a lot.

So, we are currently developing an algebra readiness program for middle schoolers in the State of Texas, and our state legislature last year allocated 55 million dollars to this project. Big chunk of money. The stakes are very high. We have estimated 130,000 kids who will fail algebra in two years if we don't turn the ship around. Very serious. So, we're beginning to work on intervention programs to work with children and bring them along, kind of improve their whole number understanding and rational number understanding. And one of the things we're battling about is this issue of algorithms.

Okay, so I need – if you're not teaching rational numbers, I need you to go back with me in your history and think about this for a minute, a little bit. We're working on proportional reasoning, so we're trying to get children to understand how to determine whether or not two ratios are equivalent. So, for the purposes of our conversation, just think of two equivalent fractions. Alright. And you remember the cross product rule. Right? If you multiply cross products and those are equal, then the two ratios are equal. Does that sound familiar to you? Okay. That cross product rule is really meaningless to children. It's a very efficient, quick way of checking whether or not the two fractions are equivalent. But it is often taught without any supporting information. And the supporting information is the fundamental fear of the fractions. The fundamental principle of the fractions that you can multiply any fraction, any number, any fraction by a number that has the same numerator and denominator, which is the same as what?

AUDIENCE: One.

DAVID CHARD: One. And you get the same fraction. Alright. So, the reason why we can do that cross product multiplication is because what you're really doing is multiplying both of them by the denominator of the other fraction and its same numerator, which is multiplying by one, to see if they're the same number. I'm not making much sense because I can't write it down for you. But the point is, is that cross product is not the algorithm. The algorithm is the fundamental principle of fractions. The cross product idea is just a quick way to get at that. And my colleagues who are working on these materials want to throw out the cross product idea altogether. Not a good idea either. Because that really doesn't encourage children's understanding. We want them to really get the whole idea behind it. So, some of this might help for you.

So, the definition, just by way of example, the definition of fraction up here, A over B plus C over D , and I want you to spend a couple of minutes looking at that, what's on the other side of that equal sign, right. What is the equivalent. Right. Really what I – I'm creating a common, what? Right. We can create a common denominator by multiplying the denominators and then because of that fundamental principle of fractions, we can multiply each numerator by the opposite denominator because it's the same as multiplying by what? By one. And that's the definition of fraction addition. If you can get that, you're done. You can do that whether you have common denominators or uncommon denominators. It still works. So, it is the fundamental definition.

What we often do then is when we teach kids to do changing mixed numbers to improper fractions, we teach this cute little idea of multiply the denominator by the whole number and add the numerator, and we should have them do 40 of those. And they have no idea why, why that works. It's very quick, it's very efficient, it works every time, but it works because of that definition. So these connections are absolutely essential. We have to show them that three - remember that concentric circle? Three is what kind of number? It's a natural number, it's a whole number, it's an integer, and because it's inside the next circle, it's also a rational number. How can three be a rational number? Because it has a numerator of one that we never write because it just takes up too much time. But, 3 is really 3 over 1. So, they have to understand that because we're going to draw on that knowledge. So three plus two-fifths is really 3 over 1 plus two-fifths. Suddenly we're right back up there to that definition and they can multiply the denominators and then multiply each numerator by the opposite denominator because that's a common denominator and that's what the definition allows us to do, and I get the same darn number. They need to be able to prove these things and work backwards.

Which is why the National Council for the Teachers of Mathematics says teaching one principle a day with 40 examples is just never going to get us where we need to be. We need to really drill down into some of these fundamental pieces like what fractions are, how they're used, what are the laws around them, and we can't just spend a few weeks each year teaching them starting in second grade and then spiraling back to them next spring and ever expect kids to really master these ideas. And they're absolutely fundamental. I keep saying that word because it's really important. If you care about your students' future, right, mathematics is going to be a critical tool they have to have.

And then the last one is you've got to get excited about this. We have to have a positive disposition about mathematics. I encourage you to spend at least one afternoon a week Googling some mathematics ideas, right. Find them. I did it in the lobby while I was waiting for the room to free up, looking for – some of you know about the laminations on the NCTM website. Google illuminations. Beautiful virtual manipulatives, ideas that you can use. They're free. We should

see - every administrator should see teachers using them in every classroom. There's the National Library of Virtual Manipulatives. Google it. And you'll find infinite numbers of manipulatives. Digital manipulatives online, on the computer, that kids can download, they can look at, they can compare, they can talk about. Because these sort of fundamental ideas have to be explored conceptually and procedurally so the kids master them.

Okay, stopping there. How's my time? Questions? Can you tell I get a little excited about it? Yeah.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Oh, yes, ma'am, absolutely. In fact, we are importing engineers as quickly as we possibly can. One of the problems we have, just as from a societal perspective, is we cannot hire international engineers to do anything in NASA or in our Defense Department, for obvious reasons, right? But that's critical. NASA is screaming for engineers. They couldn't get – can't get enough of them. And if they don't want to work in space or defense, they can work in environmental areas, water engineering. Yeah, there's an endless number of things.

It is changing very slowly, but several states have required that their teacher education programs be inspected carefully to determine exactly how much mathematics elementary teachers are getting. The models that are recommended are Michigan State and Louisiana State, where they take a series of three courses in the mathematics departments as part of their elementary education certification. Those are two states that offer elementary ed certification to undergraduates. If you Google Singapore math, you will go to their website and they will lead you to the textbooks that are used for – and by the way, I think they would be brilliant things for an elementary team in a school to study. Just to take on much like a study group for professional development for a few years. And really started working your way through – it's not complicated mathematics, but it really makes you rethink the way you understand it and the way you present it.

Yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: How old? Yeah. There's some – that's a much longer conversation, but yeah, there are a couple of really good supplementary programs. TransMath, developed by Sopris West. Very good for fourth, fifth, sixth, seventh grade. John Woodward is the author. It's a very good program. It's hard to give you without knowing the child, but –

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Absolutely. And what we've done, and I'm a special educator, right. My kids use special education. What we did is whenever kids failed to get something, we gave them a steady diet of practice and never really ensured that they understood it, and particularly for kids with learning disabilities where memory is one of the biggest challenges. They don't hold things in their short-term memory very long. Operations are very complicated. So we were just drilling operations. They would get better at them, but they never really understood them, so it did not facilitate the delayed mathematics. Yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: A number path. I don't know why they would be so significantly different than that. I mean, it seems like your idea, or the idea you've heard is useful. As long as you could ultimately transform it into the number line. Because that's what we end up using. But, if it makes sense for kids developmentally to see it as a path rather than length. I just think we have to introduce them all. Those models are all important steps. Yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: If you're talking about curricula, meaning the standards, or are you talking about the materials you publish yourself?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Well, all of them have gotten better. That's the good news. They've all started to pay attention. They're using the NCTM focal point. The State of Florida has insisted on it. So, that kind of has whipped all the publishers into shape. But I don't know that the tool itself is the problem per se. Some, really they all have, they all have the potential of introducing things incorrectly depending on who's using them or in some cases if the person who's using them isn't familiar enough with the mathematics, they can't weed out where the problems are going to be, and that's important, because no tool is perfect. Like, you can't buy a hammer and expect it to build you a house.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Absolutely.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: No, you're absolutely right. And the only study I know that has looked at that in an empirical way is one that was just finished by Mathematica that is going to continue on, but it was a second grade implementation of multiple programs and the two programs that stood out

significantly better than everything else was Satsun mathematics, which has been around for 60 years, and Math Expressions from Houghton Mifflin, which was a Katen Fuson an Eseface program. These could not be different, more different. They are at the opposite ends of the spectrum in terms of their structure, but the mathematics is accurate and they obviously helped kids conceptually and procedurally to get to the point that those kids needed to be in the achievement assessment piece of the study.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Well, my home state of Texas is doing this. We are – this summer we will have four-day-long academies for teachers at every grade level. We'll also have an end-of-course academy for teachers who teach algebra, a four through eight academy for middle school teachers who are working on understanding the rational number components. The two universities that I mentioned to you, Louisiana State and Michigan State, have really sort of taken this on by including at the pre-teaching level. And then, of course, Debra Ball and Heather Hill did their study in California. What they found, however, was that they didn't have a large enough number of teachers to determine whether or not two things: Whether or not teachers' knowledge really grew, and secondly, whether their growth was related to children's achievement outcomes. So, lots of work being done in this area. And you – others may know more, but that's what I know of right now.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Yeah, I think it's pretty good in general, yeah. I'm a big fan of the idea of the common core, but not because of the content, per se. I just see states spending millions and millions of dollars to develop standards every few years and I don't understand it. Why? Why do we do that? I worked with Native Hawaiian schools in Hawaii for 10 years, and this is a state that in many cases looks like a developing country, if you go to certain parts of the islands, and they spend 36 million dollars to develop standards and to hire a testing contractor. You could have just divvied that up amongst people and you'd be a whole lot better off, I think. So the common core makes a lot of sense to me, but if I were king, we'd also have a common national assessment, so - move on with all that stuff.

So, how am I doing time wise? Okay, I'm going to move really quickly for a few minutes.

So, let's turn our attention to student needs. Because we talk a lot about teacher knowledge and a lot about the materials and tools and such, but we also work with one of the most diverse population of kids in the world because we have compulsory education, and we're working with kids with different linguistic backgrounds, we're not – we don't have a homogenous society in that sense, so

we have to think about a number of issues, and then we have to think about the tools and practices.

So, how these things bridge. And in Pennsylvania is doing our TI, I'm assuming. I saw it in the response to intervention. Okay. So, we're all thinking about response to intervention. And that the first line of prevention is in the core classroom. How do you really help teachers be as optimally effective with the broadest range of students possible so that when kids need more, we know they really need more. It's not because they just got really bad core. And so we're trying to think of what are the, what are the fewest common challenges that kids with and without disabilities experience. And so I've borrowed from my colleagues, Scott Baker and Deb Simmons and Ed Cuminui, to look at four areas that seem to show up as common to children with learning differences, but also affect children who are not learning disabled. So, they are in these four areas. Memory and conceptual difficulties, background knowledge deficits or difficulties, linguistic and vocabulary difficulties, and strategy knowledge and use.

I like to talk to core teachers, general education teachers, about these because it kind of makes – gives us a set of finite things to think about. I don't have to imagine every unique need every child has because in a classroom of 35 kids that's impossible. So, I'm going to think about how do I create my instructions, but I can try to touch as much of this as possible and I just want to show you some examples.

First of all, how many of you regularly visit particularly school leadership? How many of you regularly visit the IDS website? The What Works Clearinghouse that we are developing – good, that's really good, very encouraged. I'm not surprised, this is Pennsylvania after all. We're developing practice guides that again would be a great source of a teacher study team, for teacher study team to look at. And this one happens to be based on some of the work that a number of us did related to the needs of struggling learners. I won't spend a lot of time here because some of you are familiar with it, but each of these practice guides summarizes the research where we know there is research around in this case children who have disabilities or are low achieving. And then they use this very handy set of these are the basic findings, and then on the side, you see these shades of green, and the darker the green, the more competence we have in the practice. So, it's a nice place to look. It's also a nice thing to think about, how does that look in the classroom? How do we observe one another as professionals, looking for this kind of behavior in the classroom?

And I'll tell you a little about how they derived these. This came from a – here's another website, by the way. Had a lot of great valuable material. If you Google centeroninstruction.org, you'll find a lot of interesting kind of supportive materials – supporting materials on different topic areas, one of which is mathematics. Some of us have contributed to this.

But we did a Meta analysis on learning disabilities and learning difficulties in mathematics. It was recently published in the *Review of Educational Research*, and this is what that guide for teachers looks like. And within that we report on a Meta analysis of about 50 studies. I know this - Lynn Fuchs is on tomorrow's agenda. Lynn – a lot of these studies are work that Lynn has done, as well as a number of other colleagues. It's a Meta analysis which allows us statistically to compare outcomes from programs across different studies, and you can read all about this, but this slide is mostly important because what it tells you is we looked at both randomly controlled studies as well as studies that were not randomly controlled, and we report effect sizes. So these effect sizes are all evidence of how much confidence we have in the finding. And anything that's bigger than .2 or .25 is noteworthy. If we were talking about prescription drug, people would be investing in it heavily, if it were to reach that level. So anything that reaches a 1., that means that the intervention actually improved children's outcomes by a full standard deviation on a standardized assessment. Okay. So it gives you a sense of how much we can say there.

So, here's the general summary of it, though. And you can look at more detail. You can also e-mail me if you have other questions. But what we found was when kids were able to verbalize and use visuals for problem solving, big outcomes, better outcomes. Now, that's not revolutionary, but it's particularly important for children in the lowest quartile, as low as 30 percent of the classroom. And if you go into core classrooms, what you often find is those doing the verbalizing and visualizing are not that group. So, it sort of raises the stakes. If I'm going to teach a core classroom and I want to do some differentiated instruction with a small group, that's what I might spend my time on; helping them verbalize. Because kids with learning disabilities, for example, are not good self talkers. Right? They don't self talk, they don't walk their way through problems. So getting them to verbalize is very important.

Secondly, explicit instruction and using specific skills. And this is one, I get blue in the face talking to some of my colleagues about this. Because they always want to start with a problem that forces kids to be inquisitive. This is a point of differentiation. Many of the kids I'm concerned about are not inquisitive. They don't know how because they don't know that they have the skills and confidence to do that. So what they do is wait for everybody else to do it. So if I want them to be as engaged in an inquiry based problem in the core classroom, I have to pre-teach it. I have to get them aside and I have to show them how to be inquisitive. I have to say – I have to show that, to model it, literally. And it can't be a remediation task where everybody else gets to do it first and then they have to do it afterwards, because that's punishment. You have to let them in on the secret. Teach them first and then say, "Tomorrow, class, we're going to come back and do a problem just like this, and these are the steps you're going to follow." And you can even lead the group.

Modification. Teachers who modify instruction based on feedback from formative assessments. So, in fact, one of the things I know Lynn is talking about tomorrow is formative assessments, the use of CBMs and such in an RTI model. This is one thing she's going to talk about. Very important to help ourselves learn how to take data, whether it's collected every week or every two weeks, and think about what did I miss rather than to continue marching through a set of few minutes without ever stopping. Why is that so important? This is one of those things I mentioned at the outset of my talk. This is different than reading. If you miss mastery of whole numbers, you will not pick it up later. It will not happen. It will be a gap that will cause you problems for life. Mathematics doesn't work that way. You can't jump into rational numbers unless you've nailed whole numbers because you will fail. It has got to be taught in a wave that allows kids to master the content as they go because of that set of concentric circles. Absolutely critical.

Alright. So, we have to teach it. We have to modify our instruction when the kids didn't get it.

And then the last piece, of course, is peer system learning. Huge effects for peer assisted learning. Given that we're not putting much energy into it as teachers. It's kids helping other kids understand concepts that they probably didn't get the first time.

Is this a good time to stop? Alright. So, we'll break for 15 minutes and be back at 3:00, and I'm going to start right at 3:00 because I have a lot more to say. Alright.

OTHER SPEAKER: And if you disappear we'll know.

DAVID CHARD: Focusing on student needs for a few minutes and talking about how to, in a tier one kind of environment, think about the needs of children who typically don't do very well mathematically, and we talked about sort of tools for thinking about the practices that are most effective for kids and those IES practice guides that demonstrate some specific things, and then I'm going to talk a little bit about instructional design related to memory and conceptual difficulties.

So, one of the things I do a lot, spend a lot of my time thinking about is really sort of information engineering because that's what we do when we plan lessons and plan units, is we're really taking what I want kids to know and how do I help them cognitively so that it increases their memory capacity and builds their understanding of what they're learning. So, we think about thoroughly developing concepts and moving from skills that are less complex to more complex, including, where we can, non-examples.

So, one of the things I often am concerned about in using, particularly as we get in the intermediate grades and into high school, is that the way we have traditionally set up textbooks is by concept or skill. And we teach it – even – we might even teach it incredibly well and then we have kids practice it, both guided and independent. Sometimes 40 or 50 times, so they're routinized the problem, the skill, and we even try to imbed it somewhat in a contrived way into a problem, but when the next day rolls around, we're done with that skill. We're moving on to the next one. And there are a couple of serious problems with that. One, kids can't retain. Even a lot of typical kids can't retain. So kids who have memory challenges are going to lose that. We also don't give them a chance to make decisions about when particular skills aren't useful, so that kind of conditional knowledge is really important. So I try to encourage people, when they assign homework, to give items where the skill that they've learned how to do can't be used. Kind of foils for them. So that they have to make decisions about whether or not the skill they've learned is actually useful in that position. And I've worked with publishers for a few years trying to get them to think about this, and it's like trying to move a mountain. It just doesn't easily happen. And then having a planful system of review.

So, here's an example of a pre-algebra lesson. By the way, I looked at my title, "Building Back on Knowledge from the Ground Up," and I probably inadvertently invited a bunch of early childhood people to my session; right? When I say building mathematics in young learners, I'm talking about K-12 education, right? Because after grade 12 we still have a whole lot of living to do. So, to me it's school, right? So, this is a pre-algebra lesson teaching kids the difference between combining like terms, and you'll notice that we're using what we hope is a somewhat meaningful example where kids are in a choir and you've got your sopranos and your altos and your tenors and your basses, and you really can't mix those, right, when they're reading music, etc., and so we're trying to make that meaningful. We're calling out the vocabulary, which I'll come back to in a moment, but we're trying to get them to understand that you can't combine those different types of singers because they're not like terms. Only the like terms can you combine. And then we move to using scaffolds to support their understanding. So, in A there, which is very simple, the terms are like, and I can understand I can combine them by adding the coefficients, and I show them how that works, right, so they see the skill that I'm trying to introduce.

You know this is a scaffolding in the second item where I circle those that are like terms with the M variable and I use a rectangle around the constants. So that they understand you can only combine those with a circle. Does that make sense? So we're building in a scaffolding system. We're also, by the way, developing – oh, here we go, here's the verbalization piece down the side, so kids can actually talk their way through the problem. Notice it gets more complicated as we go along, and then look at the last one. What have I done there? Those of you who can see it. It says $4F$ minus $12G$ plus 16 . It's one of those what? Yeah, it's a non-example. So I've taught the skill, taught the skill,

taught the skill, guide them through it, show them how to do it, model explicitly, and then I throw them a non-example. Why? I want them to know when it doesn't work. Because what do kids do? They keep adding things. Right? When they get to that homework that my 16-year-old, who just turned 17 – breaks my dad's heart. When she was in middle school, she said, "Dad, it's homework surprise every afternoon. I get home and I can't remember anything we did in class." This is adolescence, right? They don't remember much past the last three minutes. So, we really have to sort of build in these conditions so that kids see how it happens.

Then, moving to background knowledge deficits, pre-teaching, understanding children's, what they're bringing to the classroom, rather than moving in a lockstep way, assessing the background knowledge, building procedural fluency in automaticity so that they get a sense of – is that right? Did I skip a slide? That's right. Building their – providing daily opportunities for practice across skills and strategies. Frequent opportunities to respond in whole class.

Here's the message I'm trying to send. Kids need practice with these fundamental ideas, but they don't need to do 50 problems of homework every night. Give them a break. Alright. Here's what I'd prefer. Daily dose of mathematics, but not four hours daily. If you're a parent of a child in the upper grades and middle grades, honestly, I don't know how we get it done, because my kids do homework from the moment dinner ends until they fall into bed at night. It's unbelievable. And we don't need it, by the way. Have them do some of the problems in class so that we can check and make sure they're on the right track, give them 10 or so to do at night, and some of those should be things they used to already know how to do, review problems, and some of them should be new. That's enough. Then pick it back up the next day. You might also know I'm not a big fan of block scheduling in mathematics. I think it gives kids days off without thinking about mathematics. I think it's a bad thing to do. I think we need block scheduling, but five days a week. Two hours five days a week. And then it needs to be an RDA, a regular daily allotment of content.

Pre-skill testing, pre-knowledge. Here's a – these are just examples. I'm not trying to prescribe, but here's an example of an Are You Ready, so the pretest is on that side with the teacher annotations built in. I want to show you something in particular here. And this is some work that we've been doing in middle school, but can be used at any grade level. What you see is that the children's actual development or their performance on this pretest is actually linked to things you should go back and re-teach if they didn't learn it. And this is the part I was saying really we have to get serious about and revolutionize the way we do core instruction. If kids don't know the pre-knowledge necessary to be successful in the lessons you're going to teach, stop. Don't just keep moving forward. At least not with those kids. Because, you're building a house of cards. And they won't continue to get better. They will not pick it up unless you go back

and do that. So, that's going to require that we rethink the master schedule a little bit. And, of course, master schedules in schools tend to rule the day, so that requires some rethinking.

Here's what this pretest has that I really like. It has vocabulary on it. Because one thing we don't do enough of is link mathematics to language, and we're human beings. Human beings learn with language, right? By definition of our ability to speak. It also tests children's number sense. Do they have the pre-skills necessary in the numbers system that they're going to be working in, in the subsequent lessons. So, if you're going to be moving into operations with rational numbers, do they understand the fundamentals of fractions in order to do that.

And then the last piece of this is the core concepts. So, if you're going to be using rational number addition, do they know whole number addition. If you're going to be using some – if you're going to be working on decimal operations, how is their concept of place value. Do they get this. Can they compare decimal numbers.

So, you get the general jist of what – and if I were working in the first grade level, this pretest would be a whole lot shorter and I also give myself this very important luxury. This is not a test per se. Kids do not need to sit down independently and answer these questions. I think group work is the best way to pretest kids. So, have them work in groups and you have to monitor the heck out of it, right? Because you need to hear what their conversations are about. And then have them circle what they do and don't know.

I did a little sort of mini study of my own once in middle school where I – it was in a school district where kids were grouped based on need, so they had whole classrooms of children who were really high achievers and whole classrooms of kids who were average and below average. And I gave them this kind of a pretest. They worked in groups. And I said, "Just take a highlighter and mark the ones that you don't know anything about, that you really struggle with. You might have gotten right, but you really struggled with it." And, of course, the high achievers, who, by the way, didn't know a whole lot more than the average and low achievers, said they knew everything. And they didn't, by the way. The low and average achievers, though, were really hard on themselves. They were more than honest about what they didn't know. And it gave me some interesting insights into where to begin with our teaching.

So, next theory on linguistic and vocabulary difficulties. I have a three-day talk, workshop just devoted just to this topic because this is one of those areas that mathematics is extremely precise. The vocabulary is precise, the words have definitions, they're meaningful, they – we can't just throw in our own way of thinking about it without reinforcing it with the precise language, because the language we use is universal. So, it's very important for us to think about how

we introduce symbols, how we think about – one of my pet peeves is early childhood math programs that give synonyms for things that don't make any sense. You know, like we're going to do plussing today. No. I mean, five year olds can learn six-syllable dinosaur names and tell you why they're called that. They can certainly handle addition. Pretty sure. So, I don't buy into that. I think we introduce it early. We reinforce it. We help them understand why it's called that, and all the little pieces that go with it.

And then we give them the opportunity to talk about mathematics. Think about it. Discuss it. This is not a solitude kind of activity. You've got to discuss and engage in discourse. At their very earliest childhood level, we're talking about words that are what Isabelle Beck refers to as really tier one words. Those of you who have literacy backgrounds. These are prepositions, polar concepts that become really important in our understanding of how to compare numbers. For example, more than, less than, between, before, after. And then we've got vocabulary that's unique. Equal, triangle, measure, subtract, that really help us sort of hone in the kind of mathematics we're studying. And we have to give the kids a chance to talk about it. So, we have to identify what those are.

But, bear in mind, very important thing we've learned from the research done in reading and language arts, we can humanly teach in a typical kind of school about 3- to 400 words a year. Everything else children get from what they read. So that tells me two things. I have to think really strategically about what I'm going to teach and expect them to learn. What I'm going to teach explicitly. I also have to be mindful of the fact that kids need to read mathematics content. You can't escape that. And as we go up the grades, reading text about mathematics is increasingly important because when you get to post school situations, the number one source of new information is from text. It's what we reference, and using text strategically. They don't need to read from beginning to end, but they need to get into the key terms and see them used in the prose within the language. And they also need to know that no one in their right mind begins at the beginning of the page and reads to the end of the page. We dive in, we find the highlighted words, and we then ultimately have kids discuss things. Using think and discuss type items that don't always have answers. Explain why $8X + 8Y + 8$ is simplified. You don't have to simplify it. You just have to tell me why it's already simplified. Or, tell me how this is linked back to that original lesson. Describe the first step in simplifying the expression. These are the kinds of questions I love because kids always say to me, "You mean, you don't want me to do it?" No, I don't want you to do it. I just want you to tell me what the first step would be. Oh, wow. But you have to tell me. You have to say it, to describe what that first step would be, so you're going to use the distributive property. Okay, you get the point.

And then, you can go into this in more detail in my presentation online. By the way, did lots of people have trouble downloading it? Those of you who tried. Okay. Some people did. I think it's a matter of the software – you know, the

version of the software being used, but there is PDF on the website and you can find it. So, I have a plan for vocabulary mathematics. It shouldn't take up your whole mathematics lesson, but you do need to teach it.

And then the last area is strategy, knowledge, and use. Everyone in this room uses strategies every day. I drove here from – I had to find my way from Washington, D.C. to Harrisburg. I had to make plane tickets. I had to find a rental car. I had to get a map. I had to drive. All strategic thinking; right? We have to think about timing, you have to think about all the details of driving. You know what I'm talking about. We are really good at that, and we self talk our way through it and we are really good at accommodating errors. Right? Or distractions or whatever it happens to be. The children we're helping move along through their educational career are not so good at it. So we have to help them get there. And in mathematics there's a rich literature around this. There are what we call heuristics, H-E-U-R-I-S-T-I-C-S, and this is one of them where we – this is sort of a mid-level heuristic that most people use in a math classroom, where you un-try to understand the problem and you translate it into a less language intensive way of thinking about the problem. Then you translate that into numbers. That's your plan, right?

One of the things people often ask about, why it is that Hungary and Singapore and other countries do so darn well? They spend a lot of time right here. They spend a lot of time between this step and this step. Just translating problems into mathematical expressions. Because think about the job of an engineer. You come to an engineer. You want to figure out how to increase water flow to a certain region of your native lands. You're giving them a natural sort of problem in your environment. They have to figure out how to turn it into a workable mathematical problem. That's what they do. They translate it. This is an area we really have to – and, by the way, this would be a good homework problem. They don't even need to solve it. Just translate it. Right. Just spend more time translating problems from language into – from English into mathematics. Then we have them verbally rehearse the steps to solve it and look it up and see if it seems reasonable. Four steps. We should teach it very early and we should not change it. This is one of those vertical planning issues. Just because the fifth grade teacher thinks he or she has a better heuristic than the third grade teacher, changing it causes problems. And it doesn't necessarily get a whole lot better. Not measurably better. So, teaching one strategy that we use up the grades is a good idea.

There is also, for those of you who work in the upper elementary and now middle school level, some interesting research being done, both by Lynn Fuchs, and I think she will talk about this tomorrow in the morning session, but also by Asha Jitendra, who's formerly from Lehigh University and now at the University of Minnesota, where they do work on schema-based problem solving. And they have demonstrated that you can teach very successfully a lot of great elementary and middle school mathematics problem solving by helping kids really investigate

problem types. This is not key word instruction because that's distracting. This is really about looking at a problem and trying to decide what kind of structure that translates into. This is really going back to that previous heuristic where you're translating the problem into a structure. So this is just an example of some of Asha's work where she calls this a change problem. And she shows kids lots of word problems that translate into the very same change structure example. Asha has published this with Pro-Ed, so if you're looking for a way to help kids get better at making that translation, you might look at her materials at Pro-Ed. And they have – she's incredible. She's a great scientist so she has tinkered with things very carefully to make them as powerful as she can get them.

Okay. Here's my soapbox talk and then I'm going to move on to show you some findings of a specific study. But I am convinced that if we're really going to make a dent in what now people are finally realizing is a very serious situation, we have to do a number of things. We have to take responsibility for children's mathematical development. And while it's in vogue to complain about them coming to school hungry, or whatever it is we think might be affecting them, control we can control. Right? Instruction, the assessment, the time we focus on it, the enthusiasm we have, and the amount of knowledge we have. We can't control where they come from or any of those pieces. We can only control what happens in our own classroom. So, we got to get serious. Determine what we need to learn and meet our time – put our time and energy into it.

So, I do a lot of work in literacy and I tell you, people get crazy about reading. Right. Why can't we do the same thing in mathematics? Why? I think there's a lot to be passionate about here. So, let's find it. Focus our attention on the whole picture. Just improving assessments is not going to get us there. Just buying better programs is not going to get us there. We need – this is multidimensional and I showed you what my dimensions include, so sustain our effort to improve mathematics instruction for all kids. This is going to take some time. Right? It took us 40 years to get here. It's going to take us a while to get to a point where we're really graduating kids who – they may not become scientists, but they are going to feel pretty confident about what they do and how they understand math. And then we need to develop sophisticated measurement systems that better track kids' progress and do a better job of determining whether or not our instruction is being really effective. Okay, that's my soapbox.

So, I'm going to shift to a different presentation while you think of questions you have. Oh, and by the way, thank you. And I'm not going to go into great detail with this, but what time is it? 3:20, okay. Give me about 15 minutes. Yes, ma'am.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: So, I want to clarify the question. So you're saying if they are, if they are in seventh grade?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Second grade math. You should expose them to fifth grade –

AUDIENCE MEMBER: Inaudible

DAVID CHARD: It may be true. I can't refute that or confirm it. As you know, children with – who are on the autism spectrum are very mysterious about their characteristics, and we often see insights into the way they're thinking that they don't show us in their performance. So, it could be that their exposure to more high level mathematics indeed is having an impact, but we don't see it in their academic performance. Does that make sense? But I can't tell you that for sure. We don't have enough evidence of it, so this is one of the areas that people like – well, there's some work being done in reading with children on the autism spectrum. Less being done in mathematics. At a research level. But, I think it's a real valid question, and I just don't know the answer. Other – yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Yeah. Yeah. Uh-huh.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: It's a really good question. I want to make sure I don't over simplify this. So, historically, what we did is we taught the mathematics relatively procedurally. Kids with specific learning disabilities sometimes failed to do it with mastery. And then we gave them a steady daily diet of practice. And they didn't – in some cases they got a little better. Never got good enough to really employ those tools in more sophisticated mathematics. And we were absolutely holding them back. I believe that. So, in my answer to your question, here's what I want us to be clear about. Before we move them on, if they don't display mastery, I want to be darn sure they got the concept. If they could explain to me the concept, and it's some other characteristics that is preventing them from mastering the fluency piece, then move to more sophisticated conceptual mathematics and give them a tool, like a calculator, and teach them how to use it. And don't worry so much about it. But what I don't want to have happen is that we jumped over the conceptual, move to just the procedural, they failed to get it, and it was really because they didn't get the conceptual instruction that caused them to fail the procedural piece. So, if I'm satisfied that they got the conceptual instruction then, and they just are not showing improvement in fluency, then I'm going to move them forward. However, we don't have a lot of

evidence there either; right. I mean, we're really – we believe that we haven't really explored all the pieces about developing fluency, for example.

So, I want to tell you about a study we're doing. I just want to show you some of the things we're learning here, because mathematics is a very – mathematics education, particularly with struggling learners, is a pretty naïve area, and in that Meta analysis I told you about earlier, we reviewed 9,000 studies for that Meta analysis and we found 50 that met our criteria. It gives you a sense of how, how little sophistication there is in the research. So, that's a little daunting, I think.

But, my colleagues at the University of Oregon, my previous institution, and I, have been working on this program called ELM, Early Learning in Mathematics, which is a kindergarten program, and it really focuses on four areas. And we developed this program back when the NCTM focal points weren't yet even developed. So the idea of teaching less and teaching it more thoroughly was kind of novel, and we focused on numbers and operations, measurements, standard and non – or nonstandard and standard, vocabulary, and simple plain geometry, basic figures. We had a three-year federal study that developed the program and we used schools in Oregon, in high poverty areas of Oregon, Portland and Eugene, to do the initial development. Now we are in a four-year study, year two of a four-year study, to determine whether or not it works. Whether it has impact. And if so, who does it work for and what does it work to do. So, the first study was done in Oregon, and I'm going to show you those results. This year we're implementing in 66 schools in Dallas, and this includes public schools, charter schools, and parochial schools, to see what kind of impact it's having. And in Oregon, we're building a booster intervention, kind of a pre-teaching piece we call ROOTS.

So, here's the structure of the curriculum. Daily calendar lessons. So, this is kindergarten. And in Oregon we offer a half hour of kindergarten math instruction, because in Oregon you get two and half hours of kindergarten. That's it. Nobody gets more. Two and a half hours. In Texas everybody gets a full day of kindergarten. So they get an hour of mathematics. In some cases more. So, we have taken over 15 minutes of calendar, which every kindergarten teacher I've ever met does something with, and we've built calendar lessons and we've built mathematics there. It's whole class. And then we have 120 core lessons, 30 minutes of whole class, 15 minutes of teacher-directed kind of independent work with teacher support, and those 120 lessons are divided into four quarters, and then there's end of quarter assessments of progress given. Okay. That's the basic idea. I already mentioned what it's focused on. You've seen this conceptual framework and you've seen the kind of models we use. It also has a heavy dose of language. We built in the procedural fluency and automaticity that I mentioned earlier.

One of the things we do in ELM that's different than most programs I've seen is that we teach every strand every day. So, we teach number concepts and number operations every day. We don't teach it in a six-week unit and then never return to it. Every day there's some instruction in numbers. Every day there's some instruction in geometry. Every day there's some instruction in measurement. So that there's never a chance for kids, particularly kids with memory challenges, to forget something, because we cover it every day. And we build incrementally as we go. It's a pretty significant difference compared to most instruction.

And then you saw that, you saw that.

Here are our research questions. What's the immediate impact on this program when taught in general education classrooms, whole class instruction, which is what most kindergarten teachers do with some center work. On their mathematics achievement compared to a standard district practice. And the standard district practice has varied from every public program you can imagine to home grown things based on math their way. Something you might be familiar with. What's its impact, and is the impact moderated by student risk level. So if they entered with low skills on the assessments, which I'll share with you in a minute, what impact did it have on those children. And what happens when children who were not low entering, because we, in our early work, our early design work, we found kindergarteners who were doing double-digit addition in their head. Their parents didn't even know they could do those. Who knew where they got this skill. You didn't gasp at that. We were looking at the data, going, how? They were accurate every time. Fifty-seven plus 32, they'd get it right. Even with borrowing they would get it right. You knew kids who I think were on the fast track to genius.

So, I'm going to share with you the results from the Oregon study, which was done in – if you're familiar with that geography, Medford, Oregon down very south on the California border, and Portland, which is a large urban center. It was a randomized control design, so we randomly assigned teachers to treatment or control. Control being whatever they wanted to do or were doing as part of their school day, and the treatment was our program. Classrooms within school were matched on full day and half day random assignment, blah, blah, blah. We used three districts, 24 schools, 34 intervention classrooms, 30 control classrooms. A total number of students 1300, which is a pretty sizable study. Fifty-six percent of the kids were eligible for free and reduced lunch. Thirty-eight percent are English learners. You know, Oregon is quite dramatically different than a state like Pennsylvania. Oregon is about 48th in public education funding in the country, and it has some of the highest performing schools at the highest level, but increasingly large numbers of immigrants and very poorly funded school system. Ethnicity breakdowns about 50 percent, while two percent African-American, 36 percent Hispanic. That's the growing population. Some

interesting multicultural groups. Large growing population of Russian immigrants.

Here's what we expected to see happen. This is the model we expected to find. But we would teach - their instruction would be of good quality and quantity, that we would see differences in the way models of mathematics were taught, how procedural fluency was addressed in kindergarten, and the kind of conversations kids would have. Those were the three things we expected to see change compared to what teachers were doing in the comparison classrooms. And we thought we would see on the outcome of student achievements and changes.

We have a doctoral student who's doing his dissertation on classroom observations. I'm not going to spend a lot of time on this, but it's kind of interesting. We're observing in these classrooms where teachers are implementing this, and we're looking at what you're seeing - you remember the old Scantron forms? Okay. We have an observation system that uses a Scantron-like form and very sophisticated scanners that can immediately transfer it into data, and we're just looking at if the teacher modeled.

So, the black dots on the first line are whether we saw explicit teacher model, whether or not the kids responded as a whole group or individual responses. So, what we're hoping to see in the ELM program is larger numbers of modeling, larger response - kids responding, discourse, engagement. What the teacher did in terms of correcting a response. Whether there were mistakes made. The kind of feedback that was given, etc., so we're looking at some fundamental shifts in teacher behavior. I'm not going to talk to you about that as much as sort of the big outcome impact.

These were our measures. We looked at the big standardized measure, which is the test of early mathematics ability. Pro-Ed publishes this. It's a standardized 40-minute assessment. We broke it into two 20-minute assessments for the kindergarten children. And then - and we're doing that pre-post, and then we're also looking at what we call the early numeracy curriculum based measures, and there are four of those. Oral counting, so just count as far as you can for a minute. Number identification. And this is a one-minute timed - if you're familiar with Dibbles, and you know this is an Oregon study, we've created a math electables program. Number ID. So they count for a minute - they look at numbers and tell us what those are for a minute. Quantity discrimination. They're shown a series of boxes with two numbers in each box and they tell us which one's larger. And by the way, all of these were validated in a study that I published some time ago for the general learning disabilities. That particular measure of quantity discrimination, we tried every format. We tried, it doesn't make a difference if kids can point versus say the word, so if they point at eight to be larger than four, does that gives us different data than if they have to say the word eight, and in fact, saying the number is the more valid measure of

the children's knowledge. Not pointing. For whatever reason. And then we had missing number where we show kids a series of numbers and one of them is missing and they have to tell us what fits in there. So, these are all one-minute timed assessments of kids' early numeracy skill.

What I am going to show you, what I want to get to is these. So, this is time one and time two, so that's what we're measuring. That's the condition and these are their scores. I believe these are percentile scores on the TEMA, the Test of Early Math Ability. So, what you see in the solid line is children who were half – children who were in our program, and then the dotted line is half the children who were in whatever the status quo was in the classroom. A couple of observations here. It's statistically different at post-test, between the treatment and control. And at pretest they're not significantly different. So, your observations here? Overall, all kids who were getting the treatment are doing better at post-test than the group who didn't get it. That's important. One concern for us is the ES. Do you see it down there? The effect size. .14. Not that great. Which simply means the growth or the difference between them at the end is about not even a fifth of a standard deviation. Right. So, not really significant. Something to think about.

So then we look at CBM total score. So that's combining all of those conditions, all of those CBM measures and looking at total scores and seeing what happened. And what we see here, similar outcome, no different. Significant difference, almost significant, .05. That's the P value. Statistically. But the effect size is about the same. Not so great. Again, this is all kids. And jump in if you have questions.

Here is the missing number CBM score. This was total – this is missing number. Again, the P value, which if it's less than .05 is statistically significant, so that's good for getting us published, but it's not necessarily educationally important. The effect size is almost significant, getting closer to .2, which would be good. And, again, our students did better than the comparison group. Yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: That's a year, yep. Well, 120 lessons, however that got factored out, so, you know, everybody has to deal with spring assessments even though they're not doing high stakes in kindergarten, so we tried to get out of there sometime in mid-April.

So, now we're looking at risk categories. So this is kind of important. The blue kiddos are kids who entered with fairly high scores. They were above the 40th percentile. We are considering them the not at risk kids. And the kids in the red are the kids who were below that 40th percentile. The 40th percentile is sort of an arbitrary number. We could move it back and forth, but that's where we just kind of a convention in the field hit there. What do you notice? Yeah.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: No. The question is what do kids – what do teachers do when we leave in April. We just try to get the research teams out in April. The teachers keep using the program. But it is 120 lessons, so there is some wiggle room, right, in the 180-day school year. And we often don't get in there until November because kindergarten teachers are very protective of those first few weeks because kids need to learn like where the bathroom is and all that kind of stuff.

So, a couple of important things here. Mostly, the key thing here is to look at slope. What do you notice about the slope of the red lines versus the slope of the blue lines?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Exactly. This is key. And so you're looking at two things here. The control kids at risk did not grow as quickly as the treatment kids at risk, but the treatment kids at risk grew a whole lot faster than the not at risk kiddos. So, they were benefiting more from the instruction, and we noticed that over and over and over and over. So, the effect sizes are hedges G, which I just realized I told you something wrong because these effect sizes I believe anything that is around .10 is actually educationally significant. As opposed to a different kind of effect size. So, my apologies. So, we saw some significant differences here, educationally, as well as statistically.

Here's the summary. The ELM students, ELM classrooms outperformed the treatment group on the TEMA Ross score, Anderson test scores, so on standardized measures in kindergarten we saw differences between the treatment and comparison group, and on CBM measures, particularly the total and the missing number. Not on oral counting, not on number ID, and not on quantity discrimination. We believe because we've got some sort of ceiling effect. Most kids mastered that by the end of the school year.

For students with some risk, we saw some interesting things. They improved on all measures more than the no risk students. So this is not separating out treatment and control kids, just all risk kids versus all not at risk kids. And you could look at this in a couple of ways. One is, could we be moving the not at risk kids faster. I mean, why would we be satisfied that the not at risk kids aren't making more growth than the at risk kids. And that was true in the comparison and the treatment group. Yes, sir?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Well, based on what was in the measures we tested, yes, that was team one.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Correct. Didn't look at the curriculum. They didn't have a curriculum.

AUDIENCE MEMBER: Inaudible

DAVID CHARD: No, we didn't. So what you're saying is, is it possible that the kids already knew what was in the 120 lessons? Very possible, which is probably they didn't grow. But that's a question, right? I mean, that question is that should they be receiving something different. Because we always have thought that we'd give everything to kindergarten kids all the same flavor. That's what typically happens. We're now beginning to think that some kids need something else and what is it we're going to get them is the question.

So, control students at risk were catching up to not at risk students. The control kids. Want to be clear about that. But what we noticed is they grew about 14 percentage points faster than the no risk kids on TEMA, and about 9.6 percent faster than the no risk kids on CBM. The ELM kids, however, grew in comparison to the no risk kids faster; 18.6 and 20.6 on the CBM. So, there were no condition effects for students with no risk, meaning the kids at the top we didn't see difference in conditions versus treatment. So, our program did not work any better than the teacher designed or the status quo program with kids who were 40th percentile and above. Now, this is knowing who we are, this is not a big surprise because we're always worried about the strugglers, and we weren't really designing it for the upper end kids, but someone needs to ask that question; should we be moving those kids, accelerating their mathematics, learning faster.

Currently, as I said, we're implementing this in Dallas. A very different environment. Texas education starting in kindergarten textbooks. Very, very serious kind of instruction. It's very different from Oregon. It's going to be very fascinating to look at the data. We already know the teachers have covered a whole lot more content than the Oregon teachers did. So, we're going to find very different outcomes I think. And then we're doing a different kind of study in Oregon focused on the prevention piece and then we'll do that in year four in Dallas. Yes?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: Correct. That's right. Yeah. We're not – we haven't designed a higher end yet, but we're designing the lower end one to see if we

can't accelerate those kids a little bit. We suspect the teachers will come up with their own ideas about the higher end kids.

So, this is just an example of one – a research team's attempt to build a tool that we think will do a couple of things. One, assist teachers to focus on the right content; right? Less is more, go deeper, incremental development, and we're looking at whether or not it changes teachers' practices based on the content that they're receiving, particularly in kindergarten. It may be that our program is better administered in a pre-K setting. Does Pennsylvania have compulsory pre-K? Is it offered to everybody? So you're about like Texas is.

Other questions or comments about this or the other talk?

AUDIENCE MEMBER: Inaudible

DAVID CHARD: I think we've published a couple of studies on this. One was in the *Journal of Special Education*. If you Google me or Ben Clarke, C-L-A-R-K-E, I think. I think Ben's book has a patent as well. About numerous measures you'll find them. We're not the only ones who have some; other people do as well, but I think AIMSweb has a set. And by the way, there's nothing – we always laugh about it because someone said, well, have you copyrighted it? They're numbers. I don't think you can really copyright numbers, but go for it. No, we haven't copyrighted it.

AUDIENCE MEMBE: Inaudible

DAVID CHARD: Yes, there are. And we did that study in Oregon and in Austin, Texas. So we have data on fairly diverse group of – in terms of norms of expectations. Not – still need some work done in this area. If we have just two minutes, there's a lot of exciting stuff that they've done in assessment, formative assessment. My colleague Leanne Kettering Geller is working on some systems for assessing pre-algebra and Fagan at Iowa State University has developed some CBM measures for looking at algebra. Relatively easy to capture data that you can give with multiple formats over the course of the year to get at some algebra concepts. Lynn Fuchs who, of course, will be here tomorrow, has an entire system of Pro-Ed that's published on problem solving, computation and more complex problem solving. Jerry Tindle at the University of Oregon has an online system called Easy CBM for Mathematics that I believe is now being merged with Dibbles website so that you'll be able to get easy CBM and Dibbles at the same source. There's a lot of stuff that's being done. And, you know, ultimately we'll begin to coalesce around some interesting findings, but we have to be patient. Other questions or comments?

My e-mail address is my first name and my last – my first initial and my last name at SMU, Southern Methodist University dot edu. If you have questions or want to follow up discussion, please feel free to e-mail me. The first half of

this presentation is on your CD. There's a link to my website where you can download that as well. There's probably every presentation I've given in the last three years or so.

Well thank you for being here and thanks for the discussion.